Power and Sample Size Maps for Neuroimaging Studies by Non-Central Random Field Theory

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Introduction

In massively univariate analyses of neuroimaging data, determining power is a challenging task because of massive multiple comparisons among tens of thousands of correlated voxels. Although there have been some previous attempts to calculate power for such data [1-3], to date, there is no practical power calculation method controlling the family-wise error (FWE) rate. To address this issue, we propose a power analysis method for neuroimaging studies based on random field theory (RFT).

RFT-Based Power Calculation

In our method, power can be obtained by comparing the distribution of the test statistic under H0 (no signal) and under HA (signals in some areas), as seen in Fig 1. In a massively univariate analysis of neuroimaging data, the test statistic is often chosen as the maximum of a statistic image in order to control FWE. Under H0, the statistic image is modeled as a central random field (e.g., T-field, F-field) (Fig 2, area A). Under HA, the statistic image is modeled by a mixture of central and non-central random fields (Fig 2, area B). The areas of anticipated signals (Fig 2, area B) are modeled explicitly by a non-central random field.

Power to detect signals in area B (Fig 2) can be found in two steps, as described below. The resel count (μ) describes the extent of the search volume (volume, surface area, etc.). The resel density (ρ or ρ') for a random field can be found from the first and second spatial derivatives of the random field [4].

Step 1: Find the FWE-corrected threshold u0, at α-level such that

\[ \alpha \approx \sum_{i=0}^{\text{Regions}} \mu_i(A_i) \rho_i(u_i) \]

Resel count for entire brain
Resel density (for T- or F-field)

Step 2: Find power based on u0

\[ \text{Power} \approx \sum_{i=0}^{\text{Areas B}} \mu_i(B_i) \rho_i^*(u_i, \theta) \]

Resel count for area B
Resel density (for non-central T- or F-field)

Power Curve

When there is an a priori hypothesis for signal location, our method can generate a power curve to describe power to detect signals in that specific area. For example, in a simple auditory fMRI experiment, we expect activations in either of the auditory cortices (Table 1). Based on the effect sizes estimated from a pilot data analysis (Table 1), power curves can be generated, as demonstrated in Fig 3.

Power and Sample Size Maps

Power curves can be calculated for a small neighborhood around each voxel (Fig 4). Such power curves from all the voxels can be organized and displayed as a power map (Fig 5) to visualize varying sensitivity in different areas of the brain [3]. Power is calculated in a small neighborhood around a voxel, rather than at a specific voxel, in order to model the 3D nature of signals. This is because signals are often detected as a collection of voxels rather than at a single isolated voxel.

From the power curves (Fig 4), it is also possible to determine the sample size required to achieve a certain level of power. The results can be organized in the form of a 3D image as a sample size map [3] (Fig 6).

Application

We applied our power calculation method to a BOLD fMRI data set from an auditory experiment. From 41 contrast images summarizing activations associated with white noise relative to silence, 5 images were randomly selected as a mock-pilot data set and analyzed by a one-sample T-test. Although activations were found in the auditory cortices in the first-level analyses, the group analysis did not find any activation due to low df (Fig 7). Based on the mock-pilot analysis results, we generated a power map and a sample size map. We found elevated power in the auditory cortices (Fig 8, left) where signals may be detected with as few as 13 subjects (Fig 8, center). A follow-up analysis with 15 randomly selected subjects revealed activations in the bilateral auditory cortices, as predicted (Fig 8, right).

Conclusion

We developed an RFT-based power calculation method for neuroimaging studies to account for multiple comparisons among correlated voxels. With this method, power and sample size maps can be calculated from a pilot data set with relatively small N. Such power and sample size maps can visualize locally varying sensitivity in different areas of the brain, and may be useful in planning of neuroimaging studies.

References


Table 1: Areas of anticipated signals and the corresponding effect sizes for an auditory fMRI experiment.

<table>
<thead>
<tr>
<th>Areas of anticipated signals</th>
<th>Effect size</th>
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<tbody>
<tr>
<td>Left auditory cortex</td>
<td>1.15</td>
</tr>
<tr>
<td>Right auditory cortex</td>
<td>0.99</td>
</tr>
<tr>
<td>Left &amp; right auditory cortices</td>
<td>1.07</td>
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</tbody>
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Fig 1: A schematic of power calculation

Fig 2: A statistic image under H0 (left) and under HA (right)

Fig 3: An example of power curves for an auditory fMRI experiment with p<0.05, FWE corrected.

Fig 4: Power curves around different voxels

Fig 5: Examples of power maps calculated for different sample sizes

Fig 6: Examples of sample size maps for different levels of power

Fig 7: First-level and group analyses results from the mock-pilot data set.

Fig 8: A power map (left) and a sample size map (center) generated based on the mock-pilot analysis. The follow-up analysis results are also shown (right)